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Debate Concerning the Mean-Velocity Profile of a Turbulent Boundary Layer

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There has been considerable controversy during the past few years concerning the validity of the classical log law that describes the overlap region of the mean-velocity profile in the canonical turbulent boundary layer. Alternative power laws have been proposed by Barenblatt, Chorin, George, and Castillo, to name just a few. Advocates of either law typically have used selected data sets to foster their claims. The experimental and direct numerical simulation data sets from six independent groups are analyzed. For the range of momentum-thickness Reynolds numbers of 5×10^2–2.732×10^4, the best-fit values are determined for the "constants" appearing in either law. Our strategy involves calculating the fractional difference between the measured/computed mean velocity and that calculated using either of the two respective laws. This fractional difference is bracketed in the region $\pm 0.5\%$, so that an accurate, objective measure of the boundary and extent of either law is determined. It is found that, although the extent of the power-law region in outer variables is nearly constant over a wide range of Reynolds numbers, the log-region extent increases monotonically with Reynolds number. The log law and the power law do not cover the same portion of the velocity profile. A very small zone directly above the buffer layer is not represented by the power law. On the other hand, the inner region of the wake zone is covered by it. In the region where both laws show comparable fractional differences, the mean and variance were calculated. From both measures, it is concluded that the examined data do not indicate any statistically significant preference toward either law.

I. The Opening Arguments

THE Reynolds numbers encountered in many practical situations are typically several orders of magnitude higher than those studied computationally or even experimentally. High-Reynolds-number research facilities are expensive to build and operate, and the few existing are heavily scheduled with mostly developmental work. For traditional wind tunnels, additional complications are introduced at high speeds due to compressibility effects and probe-resolution limitations near walls. Likewise, full computational simulation of high-Reynolds-number flows is beyond the reach of current capabilities. Understanding of turbulence and modeling will, therefore, continue to play a vital role in the computation of high Reynolds number practical flows using the Reynolds averaged Navier–Stokes equations. Because the existing knowledge base, accumulated mostly through physical as well as numerical experiments, is skewed toward the low Reynolds numbers, the key question in modeling high-Reynolds-number flows is what the Reynolds number effects are on the mean and statistical turbulence quantities.

One of the fundamental tenets of boundary-layer research is the idea that, for a given geometry, any statistical turbulence quantity (mean, rms, Reynolds stress, etc.) measured at different facilities and at different Reynolds numbers will collapse to a single universal profile when nondimensionalized using the proper length and velocity scales. (Different scales are used near the wall and away from it.) This is termed self-similarity or self-preservation and allows convenient extrapolation from the low-Reynolds-number laboratory experiments to the much higher-Reynolds-number situations encountered in typical field applications. The universal log profile describes the mean streamwise velocity in the overlap region between the inner and outer layers of any wall-bounded flow and is the best known result of the stated classical idea.

The log law has been derived independently by Ludwig Prandtl and G. I. Taylor using mixing length arguments, by Theodore von Kármán using dimensional reasoning, and by Clark B. Millikan using asymptotic analysis. Those names belong of course to the revered giants of our field. Questioning the fundamental tenet or its derivatives is, therefore, tantamount to heresy. However, the questions and doubts linger as evidenced from the work of Simpson,[1] Malkus,[2] Barenblatt,[3] Long and Chen,[4] Wei and Willmarth,[5] George et al.,[6] Bradshaw,[7] Purushothaman,[8] and Smith,[9] among others, who at different times challenged various aspects of this law. There is strong suspicion, among the sacrilegists at least, that Reynolds number effects persist indefinitely for both mean velocity and, more pronouncedly, higher-order statistics and, hence, that true self-preservation is never achieved in a growing boundary layer. In fairness to the high priests, their log law was always intended to be a very high-Reynolds-number asymptote. These issues are discussed in greater details by Gad-el-Hak and Bandyopadhyay.[10]

More recently, Barenblatt et al.[11] and George and Castillo[12] both questioned the infallibility of the log law. To be sure, the two teams tackle the same problem quite differently and independently. Both papers offer concrete alternatives to the Reynolds number independent law of the wall. Barenblatt et al.[11] use scaling laws that invoke a zero-viscosity asymptote, whereas George and Castillo[12] introduce new tools they term asymptotic invariance principle and near asymptotic, which result in a new law of the wall with explicit Reynolds number dependence. Wosnik et al.[13] used those new tools and asserted that the inhomogeneity of a boundary layer in the streamwise direction demands separate velocity scales for the inner and outer layers. This, according to them, naturally leads to a power-law description of the overlap region. Fully developed pipe and channel flows, on the other hand, are homogeneous in the streamwise direction, and both the inner and outer flows can be scaled with a single velocity, which results in a log law but with coefficients that depend on the Reynolds number.

If in fact the classical, Reynolds number independent log law is fallible, the implications are far reaching. Flow modelers, in

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attempting to provide concrete information for the designers of, for example, ships, submarines, and aircraft, heavily rely on similarity principles to model the turbulence quantities and circumvent the well-known closure problem. Because practically all turbulence models are calibrated to reproduce the law of the wall in simple flows, failure of this universal relation virtually guarantees that Reynolds averaged turbulence models would fail too.

During the past few years, the debate between the log- and power-law camps has intensified. Advocates of either law typically have used selected data sets plotted in a certain way to foster their claims. In the present research, we use the method of fractional difference to analyze the experimental and direct numerical simulation (DNS) data sets from six independent groups for a large range of momentum-thickness Reynolds numbers. None of the data sets or the contested laws is our own, and the present analysis should provide an objective, impartial measure of the validity of either law.

II. The Debate

The controversy surrounding the validity of the classical log law has been intensified considerably during the past few years.[11–18] The decibel level of a recent shouting match (Barenblatt et al.[19] and Österlund et al.[20]) has been particularly unsettling. Entire sessions during recent AIAA, American Physical Society, and American Society of Mechanical Engineers meetings have been devoted to the controversy. The IUTAM Symposium on Reynolds Number Scaling in Turbulent Flows (Princeton, NJ, Sept. 2002), where a second shouting match took place, has been dominated by the debate.

A very simple way to derive the classical log law is to recognize the two-scale nature of the turbulent boundary layer. Close to a smooth wall, the inner region, viscosity is important, and the proper velocity and length scales are, respectively, the friction velocity u_τ and the viscous length scale (or wall unit) ν/u_τ, where ν is the kinematic viscosity and $u_\tau = \sqrt{(\tau_w/\rho)}$, where τ_w is the shear stress at the wall and ρ is the fluid density. Sufficiently far from the wall, the outer region, inertia is important there and the proper velocity and length scales are, respectively, the velocity outside the shear layer U_0 and the boundary-layer thickness δ. An overlap region is presumed to exist for distances from wall $\nu/u_\tau \ll y \ll \delta$, or roughly, and empirically, from $y = 30\nu/u_\tau$ to $y = 0.2\delta$. Note that the higher limit of this range expressed in wall units is typically a few hundreds at the typical laboratory Reynolds numbers and a few thousands at the typical field Reynolds numbers. Dimensional reasoning or asymptotic analysis yields the log law for the mean-velocity profile in the overlap region. Written in terms of the inner variables, that is, the usual $(\ )^+$ notation, the log law reads

$$u^+_{\log} = (1/\kappa)\ell n(y^+) + C_{\log} \tag{1}$$

where $u(x, y)$ is the mean streamwise velocity and the von Kármán constant κ and the intercept $C_{\log}$ are usually assumed independent of Reynolds number. The same log law can also be written in terms of the outer variables. If this log law were valid, then experimental data of the mean-velocity profile plotted in a semilog diagram should lie on a straight line over the entire overlap zone as shown generically in Fig. 1a. The normalized gradient of the velocity is given by

$$GL = \kappa\frac{\mathrm{d}u^+}{\mathrm{d}y^+} = \frac{1}{y^+} \tag{2}$$

and should be represented by a straight line with a slope of -1 in the double-log diagram shown in Fig. 1c.

The genesis of the power law is a mere curve fitting of the form

$$u^+_{\mathrm{pow}} = C_{\mathrm{pow}}(y^+)^\alpha \tag{3}$$

where the coefficient C_{pow} and the power α are generally assumed Reynolds number dependent (Schlichting[21]). Sreenivasan[22] argues that, although the power law originally used by engineers to describe the mean-velocity profile has been discredited by scientists since Millikan[23] derived the log law from asymptotic arguments, the basis for the power law is a priori as sound as that for the log law, particularly at low Reynolds numbers. Barenblatt et al.[11] and George and Castillo[12] offer competing theories to derive the power

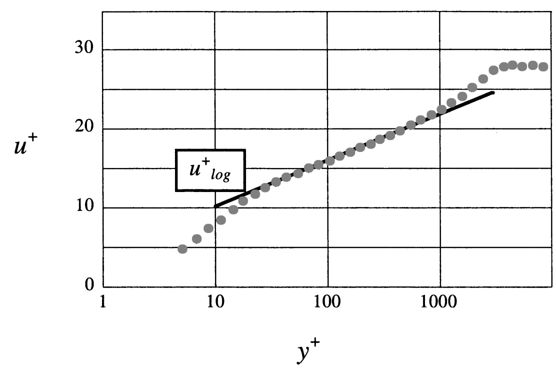

Fig. 1a Mean-velocity profile in a semilog plot.

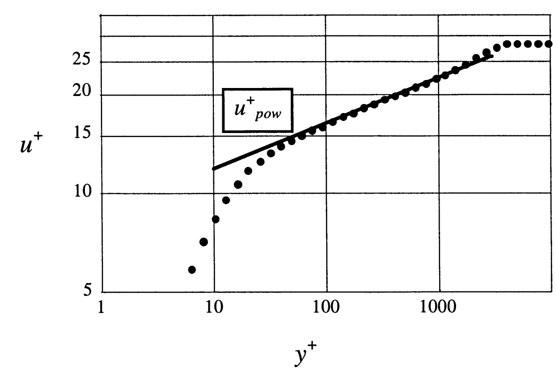

Fig. 1b Mean-velocity profile in a double-log plot.

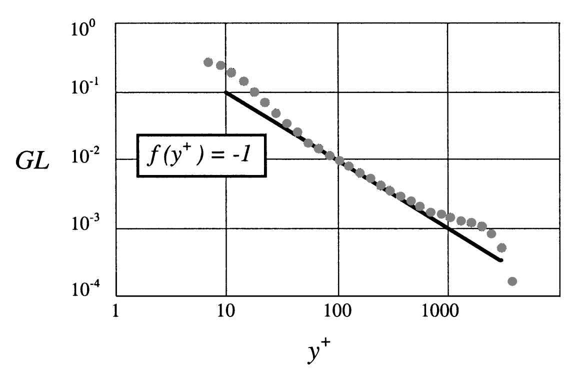

Fig. 1c Normalized gradient of the log law in a double-log plot.

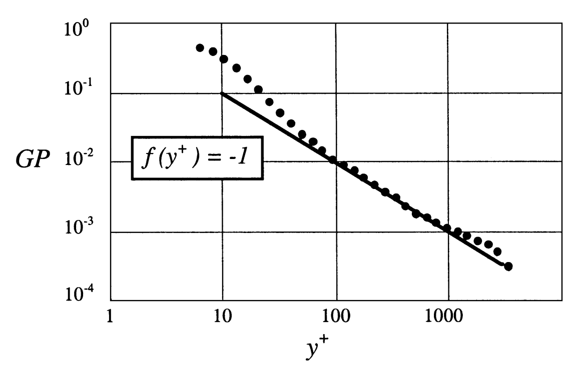

Fig. 1d Normalized gradient of the inner power law in a double-log plot.

law in the overlap region. An additional power law for the region between the outer boundary of the first power law and the outer edge of the boundary layer was suggested by Barenblatt et al.[18] To avoid confusion, the first mentioned power law will be termed inner power law and the second one outer power law. Herein, we focus on the inner power law suggested by Barenblatt.[11,18] If the power law were valid, then data concerning the mean-velocity profile should lie on straight lines in the double-log plot as shown for the inner power law in Fig. 1b. The normalized gradient

$$GP = \frac{1}{\alpha C_{\text{pow}}} \frac{1}{(y^+)^\alpha} \frac{du^+}{dy^+} = \frac{1}{y^+} \tag{4}$$

should also lie on a straight line with a slope of -1 in the double-log plot shown in Fig. 1d.

Advocates of either law typically use plots similar to those in Fig. 1 to foster their claims. The quality in relation to the way a particular law describes the mean-velocity profile has not been investigated systematically. An unbiased quality analysis applied evenly to both the log and the power laws is needed. Furthermore, if one of the laws is to be preferred, then the analysis of a large amount of independent data should show that. This is the strategy adapted herein using a rather simple tool for statistical analysis, the fractional difference, to be described next.

III. Method of Fractional Difference

The fractional difference (FD) of the mean-velocity profile is the related difference between the experimental and the theoretical value of u^+ at the same y^+ position

$$\text{FD}_{\log}\% = 100 \times \left[1 - \left(u^+_{\log} \big/ u^+_{\exp}\right)\right] \tag{5}$$

$$\text{FD}_{\text{pow}}\% = 100 \times \left[1 - \left(u^+_{\text{pow}} \big/ u^+_{\exp}\right)\right] \tag{6}$$

Analogous to this, the FD of the gradient (FDG) of the mean-velocity profile is constructed thus:

$$\text{FDG}_{\log}\% = 100 \times \left[1 - \frac{1}{\kappa} \frac{1}{y^+} \left(\frac{du^+}{dy^+}\right)^{-1}_{\exp}\right] \tag{7}$$

$$\text{FDG}_{\text{pow}}\% = 100 \times \left[1 - \alpha C_{\text{pow}} (y^+)^{\alpha - 1} \left(\frac{du^+}{dy^+}\right)^{-1}_{\exp}\right] \tag{8}$$

The FD is computed independently for all laws under consideration, regardless of which law the original authors might have contemplated. The smaller the FD is, the better the experimental data reproduced by the law applied. If one of the laws show significantly smaller FD or FDG values in a certain region of the profile, then this law should be preferred there.

Special attention is given to the determination of the gradient of the mean-velocity profile from experimental data. Most of these data are taken nonequidistantly in y^+ so that a weighted gradient formulation[24] more accurately represents the true gradient. Thus,

$$\frac{du^+}{dy^+} = \beta_1 \frac{u^+_i - u^+_{i-1}}{y^+_i - y^+_{i-1}} + \beta_2 \frac{u^+_{i+1} - u^+_i}{y^+_{i+1} - y^+_i} \tag{9}$$

where

$$\beta_1 = \frac{y^+_{i+1} - y^+_i}{y^+_{i+1} - y^+_{i-1}}, \qquad \beta_2 = \frac{y^+_i - y^+_{i-1}}{y^+_{i+1} - y^+_{i-1}} \tag{10}$$

The FD and FDG distributions are plotted for every analyzed profile individually. The best fit values for the parameters appearing in the log and the inner power law are determined. For this purpose, the FD is bracketed in the region of $\pm 0.5\%$. This procedure ensures that any a priori assumptions concerning any Reynolds number dependency of the parameters and the boundaries of the validity of the laws are excluded. The FDG distributions are used as an additional validation of the results obtained from the FD distributions.

Note that an FD of $\pm 0.5\%$ does not imply that the experimental data are accurate to within this rather narrow range. In fact, for a particular data point, the experimental error appears in both the numerator and denominator of the equation defining the FD, and hence, its direct effect is normalized out of the FD. Applied to the same data set, the FD will show a preference, if one exists, toward the log law or power law regardless of the level of experimental error. This of course assumes that the data are not totally random, and a reasonable experimental error, for example, less than $\pm 5\%$, should be readily tolerated.

The results of our analysis using the method of FD are highlighted in the next section. Complete listing of the data we used as well as the FD computations are available online at URL: <http://www.tu-dresden.de/mw/ism/sm/~forschung/TBL.htm>.

IV. The Evidence

To illustrate the method of FD, we use two data sets generated by Österlund,[15] one at $Re_\theta = 2.532 \times 10^3$ and the other at $Re_\theta = 2.0258 \times 10^4$. Österlund et al.[16] advocated a modified version of the classical log law, whereas Barenblatt et al.,[17] using the same data, advocated a power law. Herein, we compute the FD for the velocity and its gradient by seeking the best values for the constants appearing in the two respective laws. We also compute FD and FDG for the inner and outer power laws as determined by Barenblatt et al.[17] The results are shown in Fig. 2. Because of numerical differentiation, the FDG values are, as expected, considerably larger than the FD values.

When the FD distributions of the log law and the inner power law are compared, it immediately becomes clear that these laws do not cover the same y^+ region of the classical overlap zone. However, a y^+ region can be identified where both of them show similarly small FD values. In this common region, it is not possible to decide directly from the FD or FDG plots which law is to be preferred. This region will be called the common region (COR). Its extent and location obviously depend on Reynolds number Re_θ.

A small region directly below the COR is not well represented by the inner power law. Here, the log law matches the experimental data better. Therefore, this region will be called pure log region (PLR). On the other hand, a y^+ region directly above the COR is better fit by the inner power law than by the log law. For that reason, this region will be called pure power region (PPR). These findings are confirmed by the FDG plots. They are also consistent with the results of asymptotic analysis conducted by Panton.[25]

The extent of the complete power law region (COR plus PPR) partly covers the wake zone of the velocity profile. From this outcome two conclusions can be drawn:

1) Because the wake zones of a turbulent boundary layer, a pipe flow, and a channel flow are different, any relation $\alpha(Re)$ and $C_{\text{pow}}(Re)$ that has been derived from one type of flow cannot be used for another type of flow.

2) The usual approach for the wake zone, in case the classical log law is assumed, requires a wake parameter. This wake parameter is a function of Reynolds number Re_θ (for example, see Gad-el-Hak and Bandyopadhyay[10]). Any other function including the inner power law that is used for the description of this region should also display this dependency.

A truer test for the method of fractional difference is to apply the strategy to several independent data sets, none of which is our own. Six sets of mean-velocity profiles from independent teams obtained by performing experiments or by using DNS are analyzed here. The test cases include 109 velocity profiles and cover a range of momentum-thickness Reynolds numbers between 5×10^2 and 2.732×10^4. Table 1 is a summary of all of the data analyzed.[15,26–29] Note that Österlund[15] provides the most comprehensive data set extending over a very wide range of Reynolds numbers. These data are also available online at URL: <http://www2.mech.kth.se/~jens/zpg/>. The DNS data of Spalart[29] and the hot-wire results of Roach and Brierley[28] are both available in online ERCOFTAC databases at URL: <http://ercoftac.mech.surrey.ac.uk/>.

In both numerical and physical experiments, the uncertainty in measuring the mean velocity is typically better than $\pm 2\%$, whereas wall shear stress is measured or computed to within $\pm 5\%$. Österlund[15] used oil-film interferometery and reports skin-friction accuracy better than $\pm 4\%$, whereas Osaka et al.[27] used a floating element to measure directly τ and claims an accuracy of ± 1–2%. Roach and Brierley[28] utilized Clauser's method, the momentum balance approach, and a Preston tube. They report that the values of the skin friction as measured by all three methods agree to within 2%.

A word about the process by which we selected the specific data sets to be analyzed herein. First, we searched for recently published

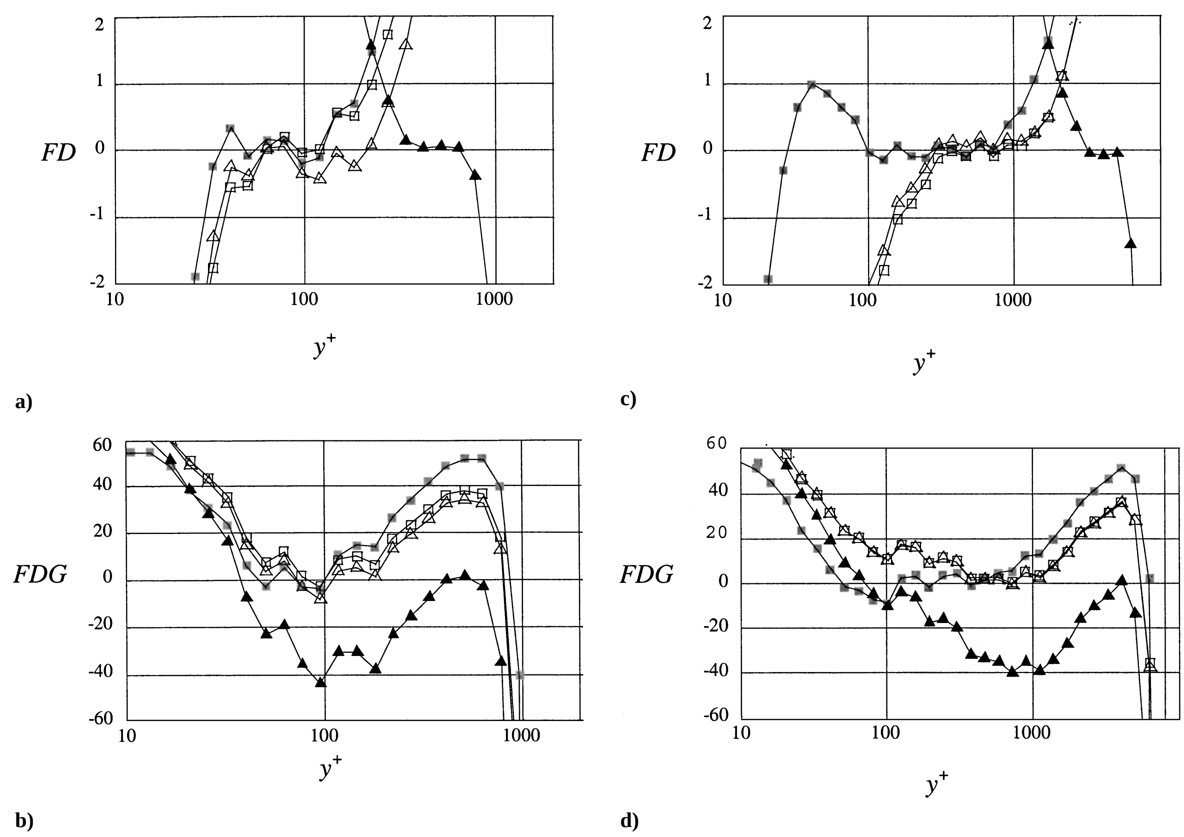

Fig. 2 Data from Österlund.[15] a) FD plots at $Re_\theta = 2.532 \times 10^3$, b) FDG plots at $Re_\theta = 2.532 \times 10^3$, c) FD plots at $Re_\theta = 2.0258 \times 10^4$, and d) FDG plots at $Re_\theta = 2.0258 \times 10^4$: ■, log law using individually determined κ and $C_{\log}$; □, power law using individually determined α and C_{Pow}; △, inner power law according to Barenblatt et al.[18]; and ▲, outer power law according to Barenblatt et al.[18]

Table 1 Analyzed sets of velocity profiles of zero-pressure-gradient turbulent boundary layers

Data set	Number of velocity profiles	Probe	Reynolds number Re_θ range, $\times 10^3$	Symbol for log law	Symbol for inner power law
Meinert[26]	6	SHW[a]	2.442–6.167	▼	▼
Österlund[15]	70	SHW	2.53–27.32	■	■
Osaka et al.[27]	14	SHW, XW[b]	0.86–6.04	▲	▲
Choi[c]	1	SHW	1.14	★	★
Roach and Brierley[28]	16	SHW	0.5–2.7	●	●
Spalart[29]	2	DNS	0.640, 1.41	◆	◆

[a]Single hot-wire probe.
[b]X-wire probe.
[c]Private communication, Sept. 1999, Udine, Italy.

boundary-layer data covering a broad range of Reynolds numbers, with an eye on the reliability and credibility of both the researchers who generated the data and the archival journals that published the results. Preference was given to experiments in which the wall skin friction was measured or computed independently of any assumption of the law governing the overlap region and to publicly available computerized data sets. Second, we excluded pipe and channel flow data, such as those from the recent superpipe experiments at Princeton University.[30] Unlike zero-pressure-gradient boundary layers, fully developed pipe flows do not continue to evolve downstream and do not possess a freestream. Last, we felt that six independent data sets were quite adequate to show all of the trends sought. Therefore, many good data sets were not included. A graph with 50 data sets will look needlessly cramped, and our selection decision by no means should be construed as a verdict against any of the excluded data, some of which are considered classical albeit unavailable in the form of computer files.

As already mentioned, no preference can be given in relation to the COR. To confirm this finding, simple statistical tests were applied. It is assumed that the FD values of each individual velocity profile have a certain probability distribution. The mean of this distribution should be zero if the correct law is applied. In that case, the scatter of the FD values is only caused by experimental error. If a wrong law is applied, the mean of the FD values will significantly deviate from zero.

In Fig. 3, the mean of the FD values of every individual profile analyzed is shown for different Reynolds numbers. Each velocity profile was fitted to both a log law and a Barenblatt-type[17,18] inner power law regardless of what the original authors have advocated in their own analysis of the same data. The key to all symbols in Figs.3–5 may be found in Table 1, gray symbols for the log-law data and black ones for the power-law data. For both laws, a remarkable collapse of the data on a horizontal line is observed in the COR (Fig. 3b). Again, no preference can be derived for one of the laws in this region. In the PLR only the log law and in the PPR only the inner power law show mean values of about zero throughout the entire Reynolds number Re_θ interval, as shown in Figs. 3a and 3c, respectively. As expected, the laws show their superiority or inferiority in their preferred region. Although not shown here, these findings are supported by the variance in the FDG distributions.

In Fig. 4, the parameters that have been derived from fitting the data to either the log law or the inner power law are plotted. All four parameters are Reynolds number dependent but to various degrees. Whereas the coefficient C_{pow} and the power α are Reynolds number dependent throughout the entire Reynolds number interval investigated, the parameters of the log law seem to reach constant values with $\kappa = 0.384$ and $C_{\log} = 4.171$ above $Re_\theta \approx 10 \times 10^4$. These asymptotic values are very close to the values found by Österlund et al.,[16] who advocated a log law but only above $y^+ \approx 200$ and for Re_θ exceeding 6×10^3 (so that a measurable overlap region could exist). The Österlund et al. values are $\kappa = 0.38$ and $C_{\log} = 4.08$. However, it is also found that the α and C_{pow} values derived here from Österlund's[15] data are in good agreement with the equivalent values derived directly by Barenblatt et al.,[17] who fitted the same data to an inner power law.

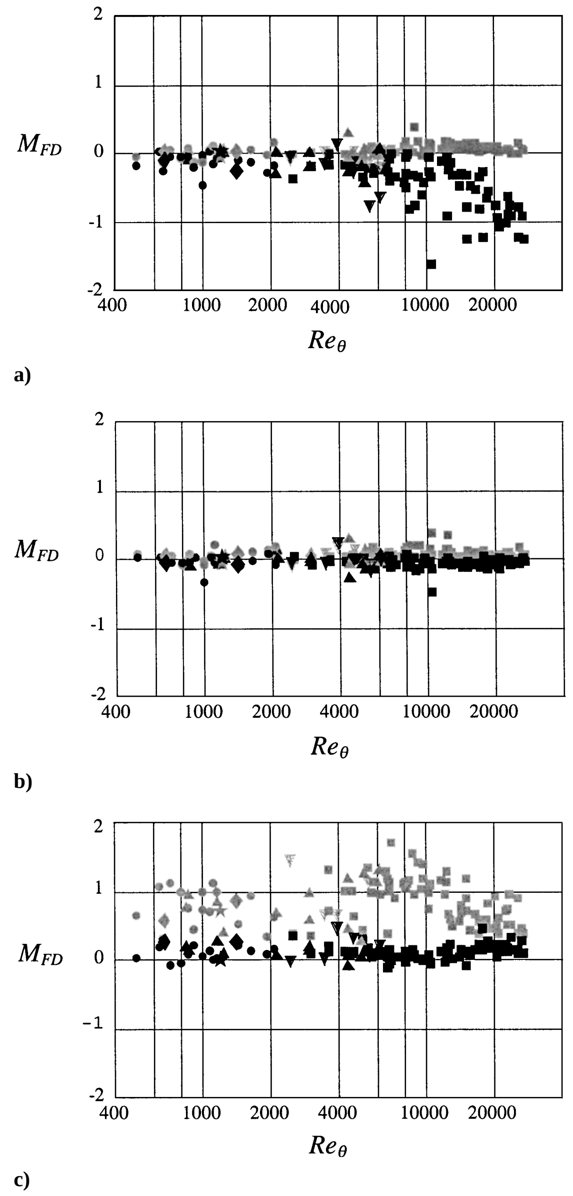

Fig. 3 Mean of FD distributions (see Table 1) for a) PLR, b) COR, and c) PPR.

It is very interesting to compare the boundaries and extent (the distance between the inner and outer boundary where FD does not exceed ±0.5%) of both the log law and the inner power law. These are shown in Fig. 5, both in inner and outer variables, as function of the momentum-thickness Reynolds number. In Figs. 5a and 5c, the boundaries of the log region (PLR and COR) and the power region (COR and PPR) are emphasized in inner and outer variables, respectively. In Figs. 5b and 5d, the extent of both regions are emphasized in inner and outer variables, respectively. The boundaries in inner variables clearly indicate that the region where the log law or power law is valid is located at different positions within the profile. Almost throughout the entire Reynolds number Re_θ interval, the boundaries of the inner power law are located above the corresponding boundaries of the log law (Figs. 5a and 5c). All boundaries move away from the wall as the Reynolds number increases. However, this movement is different for both laws above $Re_\theta \approx 4 \times 10^3$. Whereas the boundaries of the inner power law converge, the boundaries of the log law diverge. The reason for the latter is that the outer boundary of the log law grows faster than the corresponding boundary of the inner power law. On the other hand, the inner boundary of the log law grows only very slowly, whereas the inner boundary of the inner power law increases rapidly. The extent of the log region (PLR plus COR) clearly reflects this behavior. Although it is smaller than the extent of the inner power law (COR plus PPR), the log-law region is growing faster than the power-law region (Fig. 5b).

Figures 5c and 5d using outer variables indicate a more complicated picture. The inner boundaries of both laws are almost the same for $Re_\theta < 1 \times 10^4$. Above this Reynolds number, the inner boundary of the log law becomes almost constant, whereas the corresponding boundary of the inner power law starts to move away from the wall. For $Re_\theta < 4 \times 10^3$, the outer boundaries of both laws move toward the wall as the Reynolds number increases. When the Reynolds number Re_θ exceeds 4×10^3, the outer boundaries move away from the wall. Whereas this tendency lasts for the log law up to the highest Reynolds number investigated, the outer boundary of the inner power law becomes almost constant for $Re_\theta > 1 \times 10^4$. The effect of this behavior is that the extent of the log law keeps growing, whereas the power law shows a constant extent (Fig. 5d).

From classical theory, it is known that the log law should be the asymptotic behavior of all inner layers.[25] When it is assumed that the inner power law describes the inner layer of the boundary layer, then the envelope of all individual curves of the inner power law should be a log function. The existence of such an envelope was first shown by Barenblatt.[14] This can be checked by calculating the envelope using the computed $C_{\text{pow}}(Re_\theta)$ distribution and $\alpha(Re_\theta)$ distribution. To do this, the inner power law is rewritten as follows:

$$F\left(y^+, u^+, Re_\theta\right) = C_{\text{pow}}(Re_\theta) \times (y^+)^{\alpha(Re_\theta)} - u^+ \qquad (11)$$

The points of the envelope (y_E^+, u_E^+) are then obtained from

$$F\left(y^+, u^+, Re_\theta\right) = 0, \qquad \frac{\partial F\left(y^+, u^+, Re_\theta\right)}{\partial Re_\theta} = 0 \qquad (12)$$

by eliminating Reynolds number Re_θ.

Data from Österlund[15] were used for this test. After very carefully smoothing the $C_{\text{pow}}(Re_\theta)$ and $\alpha(Re_\theta)$ distributions, the gradient formulation for nonequidistant discrete distributions [Eq. (9)], was applied to obtain the gradients

$$\frac{\partial \alpha(Re_\theta)}{\partial Re_\theta}, \qquad \frac{\partial C_{\text{pow}}(Re_\theta)}{\partial Re_\theta} \qquad (13)$$

All pairs (y_E^+, u_E^+) that were found outside the accompanying individual power region (COR plus PPR) were eliminated as erroneous. From the original 70 velocity profiles, 56 pairs of (y_E^+, u_E^+) were obtained and compiled in Fig. 6. Indeed, these points follow a straight line in the semilog plot. A corresponding curve fit leads to the constants $\kappa_E = 0.365$ and $C_E = 3.339$. A comparison of the log law obtained by Österlund et al.[16] and the envelope shows that they lie very close together, as indicated in Fig. 6. Nevertheless, note that the log law and the envelope found here are not a priori identical. Whereas the log law is a description of the mean-velocity profile derived from a limiting process matching inner and outer series expansions, the envelope is a mathematical feature of the power law derived from experimental data. The log law is obtained for large but finite Reynolds numbers. The envelope is valid for the entire range of Reynolds numbers investigated herein.

The FD and FDG plots shown in Fig. 2 also show results for the outer power law according to Barenblatt et al.[18] For these distributions, only the original parameters (power and coefficient) given by Barenblatt et al. were used. The evidence that the inner and outer power laws do not match comes mainly from the FD plots. A significant y^+ gap exists between these laws. This gap cannot be seen if only the usual semilog plot is utilized. Moreover, only three or four points are covered with the outer power law. When it is kept in mind that this power law has two degrees of freedom, then at least two points of the mean-velocity profile should be captured with high accuracy. A third or a fourth point in between those two or next to them may be captured accidentally. These features may indicate that the outer power law, as conceived by Barenblatt et al.[18] is a kind of curve fit with no physical universality.

V. The Verdict

An impartial analysis concerning the modeling of the mean-velocity profile of the canonical turbulent boundary layer was undertaken. Based on an objective evaluation of experimental and DNS

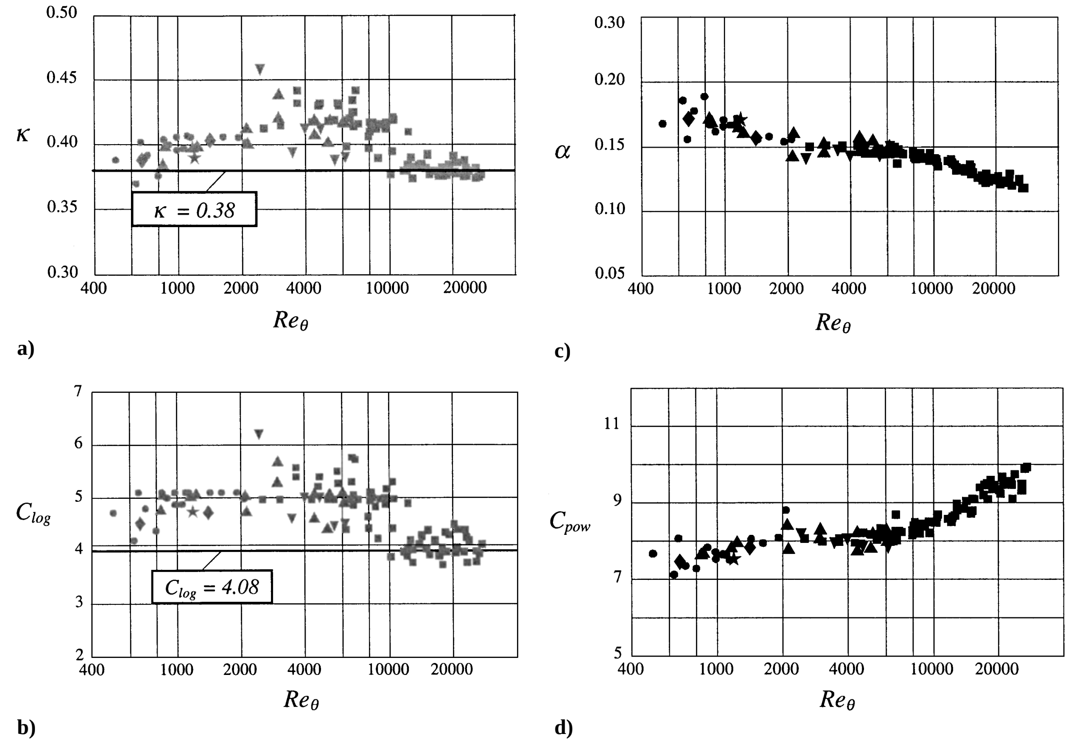

Fig. 4 Parameters of log law and power law obtained by using FD method (see Table 1); indicated average values for κ and $C_{\log}$ computed by Österlund et al.[16]

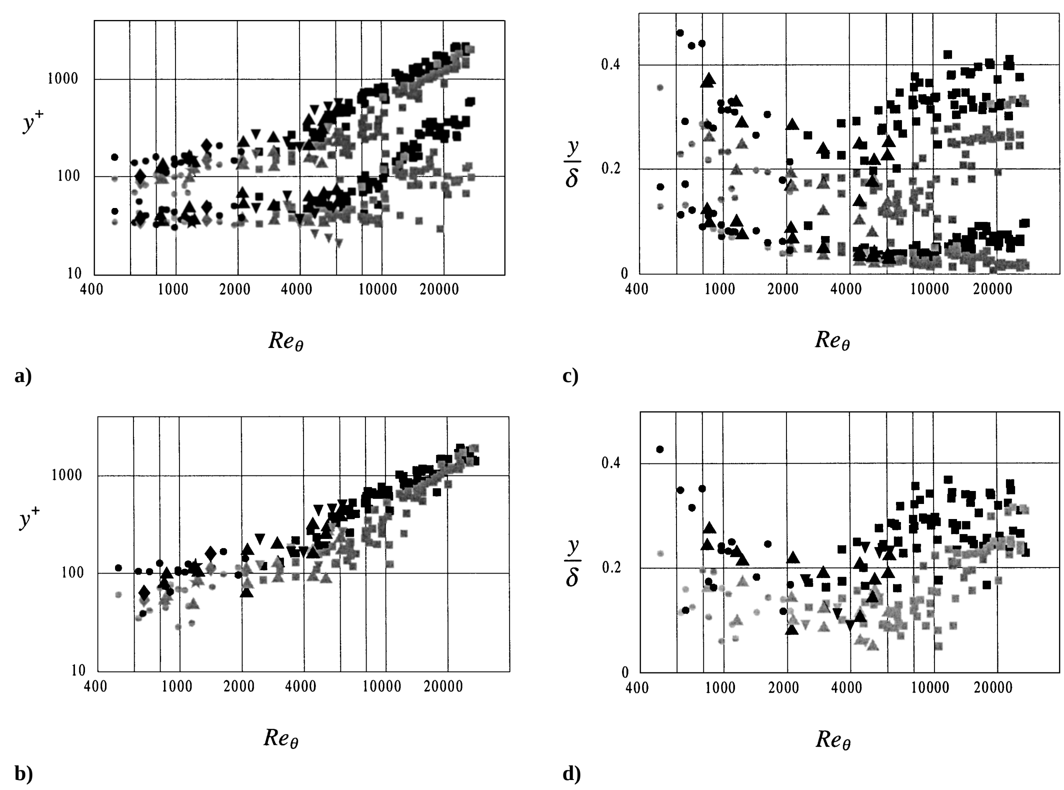

Fig. 5 Boundaries and extent of log region and power region (see Table 1): a) boundaries of log region (PLR and COR) and power region (COR and PPR), dimensionless *y* in inner variables; b) extent of log region and power region, dimensionless *y* in inner variables; c) boundaries of log region and power region, dimensionless *y* in outer variables; and d) extent of log region and power region, dimensionless *y* in outer variables.

results, arguments of supporters of the log law on one side and supporters of the power law on the other side were examined. To ensure maximum objectivity, no data taken by the present authors were included. The results are summarized in the following four points and schematically shown in Fig. 7:

1) Neither the log law nor the inner power law is valid throughout the entire overlap region. The examined data do not indicate any statistically significant preference toward either law in the common region where the FDs of the mean-velocity profile for the log and power laws are of the same order.

2) Below the COR, a zone exists where the log law reproduces the experimental data better than the power law. Above the COR, a zone is found where the power law is in better agreement with the experimental data.

3) The envelope of the individual curves of the inner power law is a log function. The COR mentioned earlier can be understood

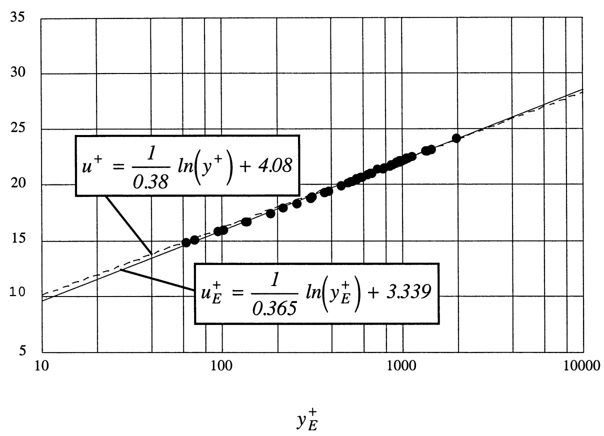

Fig. 6 Envelope of individual power law curves of the data set by Österlund[15]: ———, envelope and - - - -, log law according to Österlund et al.[16]

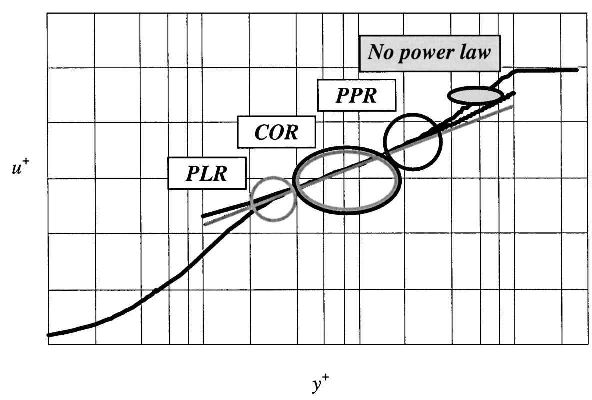

Fig. 7 Schematic of different regions of mean-velocity profile of canonical tubulent boundary layer: gray line, log law; and black line, inner power law.

as the region where the inner power law departs from this envelope.

4) There is no second power law region in the wake zone of the profile. A much better fit of the data in the outer region of the profile can be obtained by using a wake function that is added to the inner power law to complete the profile.[12,31]

Using Poincaré expansions and asymptotic matching, Buschmann and Gad-el-Hak[32] recently advanced a generalized log law that includes higher-order terms involving the von Kármán number and the dimensionless wall-normal coordinate. The new law is theoretically supported by the Lie-group analysis of Oberlack.[33] Buschmann and Gad-el-Hak convincingly demonstrate the superiority of the generalized log law over the classical log law or the Reynolds number dependent power law.

How should an experiment be designed to learn more about the correct description of the mean-velocity profile? We offer five suggestions:

1) It is not necessary to go to higher and higher Reynolds numbers because all effects can already be seen using profiles with moderate Re_θ of about 1×10^4.

2) The mean-velocity profile and the wall skin friction should be measured independently and directly.

3) The experimental technique used should make allowances for the fact that the difference between the log law and the power law is very small. It is found here that Δu^+ is about 0.4 ($\Delta u^+/u^+ \approx$ 1–2%) for a profile with $Re_\theta \approx 1 \times 10^4$, when PLR, COR, and PPR are taken into account.

4) To ensure that wall-normal gradients can be computed with high accuracy, the mean profile should be taken equidistantly in y.

5) To allow more sophisticated statistical analysis, as much data as possible should be taken in the region of interest.

In closing, existing data covering a wide range of Reynolds numbers support the log law and power law with equal measure throughout most of the overlap region. Advocates of either law should look elsewhere to resolve their conflict, peacefully. This court is now adjourned.

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